

A polynomial function $g(x)$ of degree 5 has a zero of $2 - i$ and a zero of $4 + i$. What is the maximum number of real zeros for $g(x)$?

1. (A) 0 (C) 2
(B) 1 (D) 3

If $f(x) = 2x + 4$ and $g(x) = x^2 - 2$, then $(f \circ g)(x)$, where $(f \circ g)(x)$ is a composition of functions, is

2. (A) $2x^2 - 8$ (C) $2x^2$
(B) $2x^2 + 8$ (D) $2x^3 + 4x^2 - 4x - 8$

Each point of the graph of $f(x) = (x + 3)^2 - 5$ is moved 4 units in a positive horizontal direction and 6 units in a negative vertical direction. If this new graph is called $g(x)$, which of the following describes $g(x)$?

3. (A) $g(x) = (x - 1)^2 - 11$
(B) $g(x) = (x - 1)^2 + 1$
(C) $g(x) = (x + 7)^2 - 11$
(D) $g(x) = (x + 7)^2 + 1$

The equation $x^2 + 2x + 7 = 0$ has

4. (A) two complex conjugate roots.
(B) two real rational roots.
(C) two real equal roots.
(D) two real irrational roots.

What is the equation of the line that is parallel to $6x + 3y = 4$ and has a y -intercept of -6 ?

- (A) $y = 2x - 6$ (C) $y = -2x - \frac{4}{3}$
5. (B) $y = 2x + \frac{4}{3}$ (D) $y = -2x - 6$

Which of the following functions has its highest point at $(-1, 5)$?

- (A) $f(x) = -2(x - 1)^2 - 5$
(B) $f(x) = 2(x + 1)^2 - 5$
(C) $f(x) = -2(x - 1)^2 + 5$
6. (D) $f(x) = -2(x + 1)^2 + 5$

Consider the function $F(x)$ defined as follows:

$$F(x) = \begin{cases} -x + 3, & \text{if } x < 0 \\ 2x + 3, & \text{if } 0 < x < 3 \\ 3x, & \text{if } x > 3 \end{cases}$$

What is the value of $F(-3) + F(2) - F(4)$?

- (A) 2 (C) -1
7. (B) 1 (D) -2

Consider the function $F(x)$ defined as follows:

$$F(x) = \begin{cases} -x + 3, & \text{if } x < 0 \\ 2x + 3, & \text{if } 0 < x < 3 \\ 3x, & \text{if } x > 3 \end{cases}$$

What is the value of $F(-3) + F(2) - F(4)$?

- (A) 2 (C) -1
(B) 1 (D) -2

8.

Find the determinant of the matrix A where:

$$A = \begin{bmatrix} 2 & 7 & -3 & 8 & 3 \\ 0 & -3 & 7 & 5 & 1 \\ 0 & 0 & 6 & 7 & 6 \\ 0 & 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

- (A) 90
(B) -1,296
(C) -1008
(D) -90

9.

$$\text{Let } A = \begin{bmatrix} 2 & 3 & 7 \\ 4 & m & \sqrt{3} \\ 1 & 5 & a \end{bmatrix}, \quad B = \begin{bmatrix} \alpha & \beta & \delta \\ \sqrt{5} & 3 & 1 \\ p & q & 4 \end{bmatrix}$$

Find $A + B$

- (A) $\begin{bmatrix} 2+\alpha & 4+\sqrt{5} & 7+\delta \\ 3+\beta & 1+p & \sqrt{5}+q \\ 7+p & 3+\beta & a+4 \end{bmatrix}$
(B) $\begin{bmatrix} 2+\delta & 3+\alpha & 7+\beta \\ 4+1 & m+1 & \sqrt{3}+\sqrt{5} \\ 1+4 & 5+4 & p+4 \end{bmatrix}$
(C) $\begin{bmatrix} 2+\alpha & 3+\beta & 7+\delta \\ 4+\sqrt{5} & m+3 & \sqrt{3}+1 \\ 1+p & 5+q & a+4 \end{bmatrix}$
(D) $\begin{bmatrix} 2+\alpha & 3+\delta & 7+\beta \\ 4+3 & m+\sqrt{5} & \sqrt{3}+1 \\ 1+q & p+5 & a+4 \end{bmatrix}$

10.

Find the determinant of the following matrix:

$$A = \begin{bmatrix} 2 & 0 & 3 & 0 \\ 2 & 1 & 1 & 2 \\ 3 & -1 & 1 & -2 \\ 2 & 1 & -2 & 1 \end{bmatrix}$$

- (A) 11
(B) 15
(C) 19
(D) 36
- 11.

In 3-dimensional space, let $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$,
and $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, represent three vectors. Show
why the vector $\begin{pmatrix} 5 \\ 6 \\ 9 \end{pmatrix}$ *cannot* be written as a linear

12. * combination of α_1 , α_2 , and, α_3 .

Use the principle of mathematical induction to prove that:

$$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

for all positive integers n .

13. *

14. * Given the equations $546x + 26y + 243z = 436$, $961x + 164y + 188z = -24$, and $-12x + 362y - 288z = 582$.
- Represent the system of equations as a product of matrices.
 - Describe how to use the equation in part (a) to determine the solution (Do not solve).

15. * Let $f(x) = x^4 - x^3 - 7x^2 + bx + c$ and $g(x) = x^2 + 2x - 3$, where b and c are constants, then if $g(x)$ is a factor of $f(x)$:
- Find the values of b and c .

b. If $h(x) = \frac{f(x)}{g(x)}$, find $h(x)$ and tell how many roots it has.

c. Tell the sets of numbers the roots of $h(x)$ belong and list the roots.

d. Find all the roots of $f(x)$.

16. * $f(x) = 4x^3 - 9x$ $g(x) = \frac{1}{f(x)}$ $h(x) = \sqrt{f(x)}$

- Sketch $f(x)$ indicating intercepts.
- Sketch $g(x)$ indicating asymptotes.
- Sketch $h(x)$ indicating intercepts.

17. * Given $x^5 - 2x^4 + 2x^3 - 11x^2 - 8x - 12$

a. Find all the rational roots.

b. Given that $2i$ is a root of the polynomial, write the polynomial as a product of a linear equation and two quadratic equations.

c. Find all of the complex roots in the form of $a + bi$.

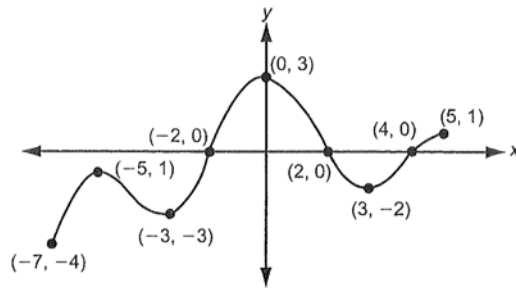
18. * Use math induction to prove $4^n - 1$ is divisible by 3 for all natural numbers.

19. The sum of all positive integers from 1 to n is: $\sum_{k=1}^n k = \frac{n(n+1)}{2}$

a. Find the sum of the integers from 1 to 100.

b. Find the sum of all the even integers from 1 to 1000.

c. Prove that the formula for the sum of the even integers from 1 to n is $\frac{n(n+2)}{4}$.

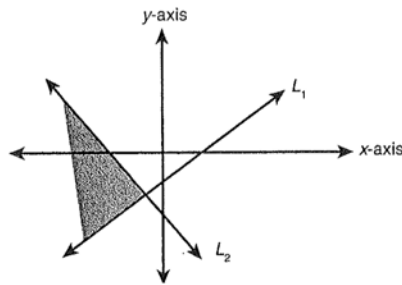


The graph of $g(x)$ is shown above. How many zeros would exist for the graph created by shifting $g(x)$ vertically downward by two units?

20. (A) 0 (C) 2
(B) 1 (D) 3

The height of an object is given by the equation $z = -16t^2 + 144t$, where z is the distance in feet and t is the time in seconds. After how many seconds will this object reach its maximum height?

- (A) 3
(B) 4.5
(C) 7.5
21. (D) 9



The shaded region could represent the graphical solution to which pair of inequalities?

- (A) $2x + 3y \geq -6$ (C) $2x + 3y \leq -6$
 $x - y \leq -1$ $x - y \leq 1$
 (B) $2x - 3y \geq 6$ (D) $2x - 3y \leq 6$
 $x + y \geq -1$ $x + y \geq 1$

22.

Define the symbol \spadesuit as follows:

$$\text{if } x < y, x \spadesuit y = y^3 - 3x$$

$$\text{if } x \geq y, x \spadesuit y = x^2 + y^2$$

What is the value of $(2 \spadesuit 3) \spadesuit 4$?

- (A) 403
 (B) 425
 (C) 457
 (D) 481

23.

A function contains the points $(8, 3)$, $(-1, -3)$, $(-2, 6)$, and $(7, 6)$. Which one of the following *must* be a point belonging to the inverse of this function?

- (A) $(3, -8)$
 (B) $(1, 3)$
 (C) $(-2, 7)$
 (D) $(6, 7)$

24.

Find $A + B$ where $A = \begin{bmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \end{bmatrix}$,

$$B = \begin{bmatrix} 0 & 2 & -4 \\ 1 & 3 & 1 \end{bmatrix}$$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ (C) $\begin{bmatrix} 3 & 0 & 4 \\ 1 & 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 0 & 0 \\ 1 & 2 & 4 \end{bmatrix}$

25.

If the graph of $px - 4y = 12$ is perpendicular to the graph of $5x + 6y = 24$, what is the value of p ?

(A) 5.4

(B) 4.8

(C) 3.75

26. (D) 3.33

Consider the function $f(x) = mn x^2$, where m and n are constants. Which of the following sequences is geometric?

(A) $f(2), f(6), f(8)$

(B) $f(1), f(2), f(3)$

(C) $f(2), f(4), f(8)$

27. (D) $f(3), f(5), f(7)$

Which one of the following functions has no horizontal asymptote?

(A) $f(x) = \frac{(3x^2 + x - 7)}{(4x^2 + x + 1)}$

(B) $f(x) = \frac{(x - 4)}{(5x^3 + 3x - 1)}$

(C) $f(x) = \frac{(x^4 + x^3 - x + 3)}{(x^5 - x^4 + 2)}$

(D) $f(x) = \frac{(x^3 + 7x^2 - 2)}{(2x^2 - 5)}$

28.

A vector \mathbf{v} is described by $\langle 5, -12 \rangle$. Which of the following describes a vector of length $\frac{1}{2}$ unit in the direction of \mathbf{v} ?

(A) $\left\langle \frac{5}{26}, -\frac{6}{13} \right\rangle$

(B) $\left\langle \frac{10}{13}, -\frac{24}{13} \right\rangle$

(C) $\left\langle \frac{5}{7}, -\frac{12}{7} \right\rangle$

(D) $\left\langle \frac{5}{2}, -6 \right\rangle$

29.

Which one of the following matrices represents

the product of $\begin{bmatrix} 2 & 3 & -1 \\ -2 & 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 \\ 2 & 0 \\ -1 & 2 \end{bmatrix}$?

(A) $\begin{bmatrix} 9 & -2 \\ 4 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} 9 & 4 \\ -2 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} -9 & -4 \\ 2 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} -9 & 2 \\ -4 & 2 \end{bmatrix}$

30.

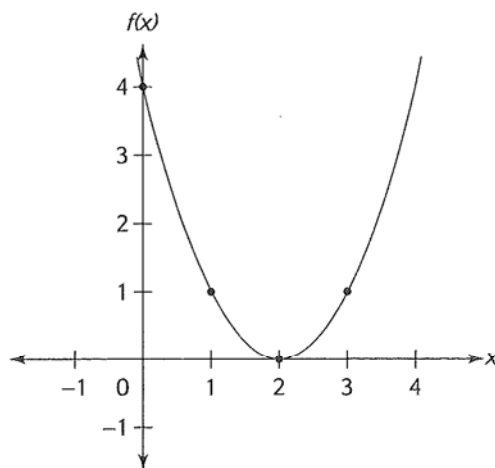
Let $f(x) = x^2 + cx + d$, where c, d are real numbers. Suppose $f(x)$ has a real zero at $1 + i\sqrt{2}$.

- Determine the values of c and d .
- Determine the y -intercept and the location of the vertex.

31. *

In 3-dimensional space, let $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ and $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ represent three vectors. Show that the vector $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ is a linear combination of $\alpha_1, \alpha_2, \alpha_3$.

32. *



Which of the following is the equation of the graph shown above?

- A. $y = 2x^2 + 4$
- B. $y = x^2 - 4x + 4$
- C. $y = x^2 + 4x + 4$
- D. $y = x^2$

33.

Identify whether one of the following vectors is equal to \overline{MN} if $M = (2, 1)$ and $N = (3, -4)$:

\overline{QR} , where $Q = \langle -4, 5 \rangle$ and $R = \langle -3, 10 \rangle$

\overline{LP} , where $L = \langle 1, -1 \rangle$ and $P = \langle 2, 3 \rangle$

\overline{ST} , where $S = \langle 3, -2 \rangle$ and $T = \langle 4, -7 \rangle$

- A. Only \overline{LP} is equal to \overline{MN} .
- B. Only \overline{QR} is equal to \overline{MN} .
- C. Only \overline{ST} is equal to \overline{MN} .
- D. None of these three vectors are equal to \overline{MN} .

34.

Identify which of the following is a negative number.

- A. i^{32}
- B. i^{25}
- C. i^{50}
- D. i^{75}

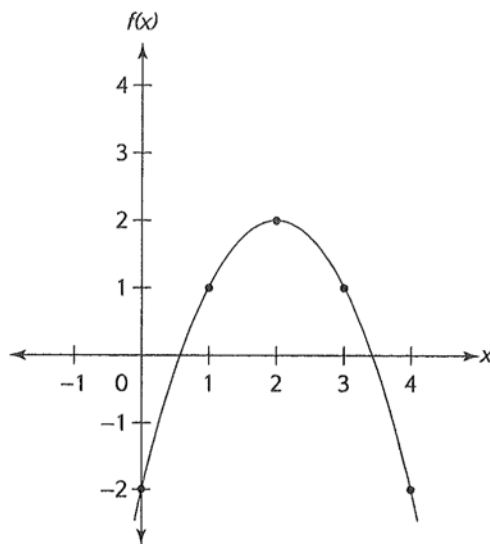
35.

Assuming that $i^x = i^y$, which of the following alternatives is or are always true?

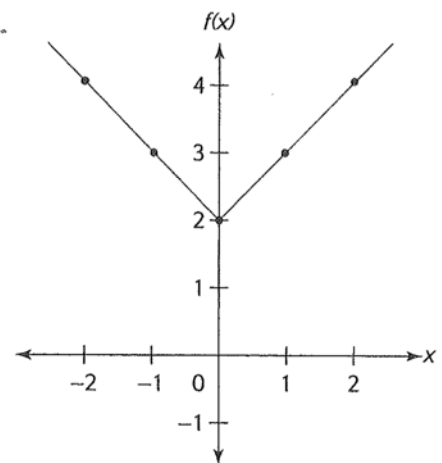
- I. x is equal to y .
- II. x plus y is equal to an even number.
- III. $x - y$ is a multiple of 4.
- A. Only I is true.
- B. Only III is true.
- C. Only I and III are true.
- 36. D. Only II and III are true.

Identify the graph that matches the following equation: $y = |x^2 - 4x + 2|$

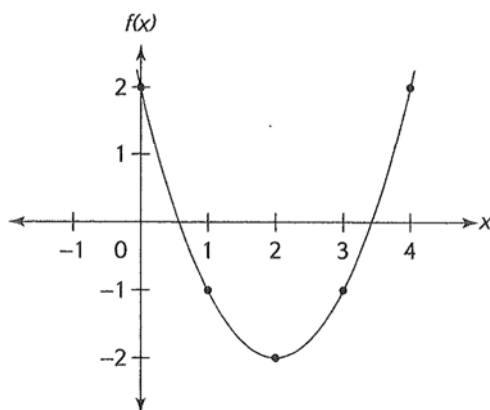
A.



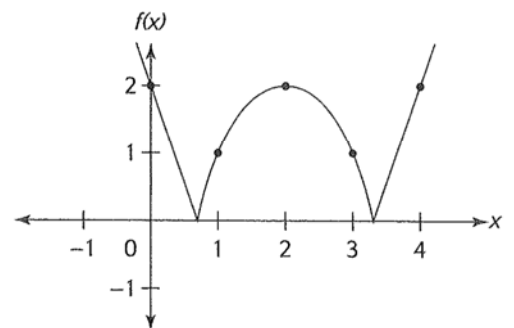
C.



B.

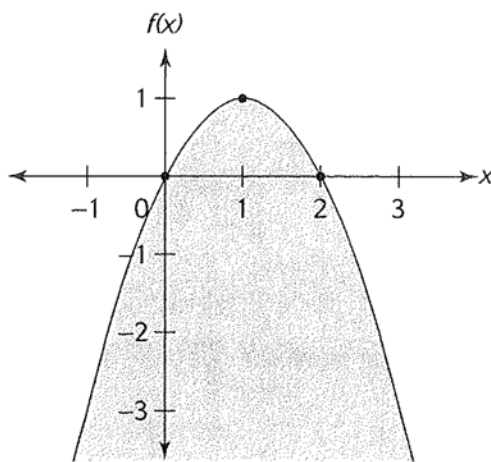


D.



37.

Use the graph below to answer the question that follows.



Which one of the following inequalities describes the shaded region shown in the graph?

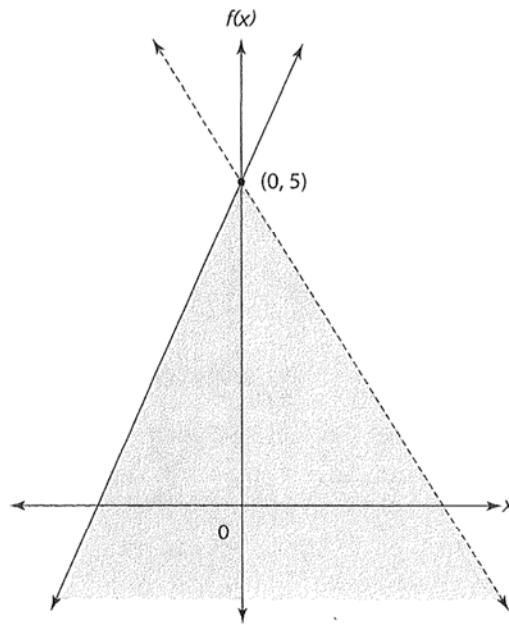
38. A. $y \leq -x^2 - 2x$
 B. $y > -x^2 + 2x$
 C. $y \leq -x^2 + 2x$
 D. $y \leq -x^2 + 2x + 2$

Which pair of vectors is perpendicular?

39. A. $\langle -2, 1 \rangle, \langle 1, -2 \rangle$
 B. $\langle -2, 1 \rangle, \langle -1, -2 \rangle$
 C. $\langle 2, -1 \rangle, \langle -1, 2 \rangle$
 D. $\langle -2, -1 \rangle, \langle -1, -2 \rangle$

Burger

Use the graph below to answer the question that follows.



40.

Which of the following systems of inequalities represents the shaded region above?

- A. $y - 2x \leq 5$
 $y + x < 5$
- B. $y - 2x \leq 5$
 $y + x \leq 5$
- C. $y - 2x \geq 5$
 $y + x < 5$
- D. $y - 2x < 5$
 $y + x > 5$